Engineering Experiment Station

University of Illinois

Technical Memorandum No. 5

CROSS-WIRE PROBE FOR HOT-WIRE ANEMOMETER

by

J. E. Romano and H. F. Johnstone

Issued in Conjunction with the Final Report on the Program

"Mixing of Fluid Streams"

Contract No. N6-ori-071(11)

Office of Naval Research, Department of the Navy

Urbana, Illinois

September 30, 1953

Distribution List

Copies	Government Agencies	Copy No.
15	ONR, Washington, Attn: Fluid Mech. Br. (Code 426)	1-15
8	Biological Dept., Camp Detrick, Attn: Dr. J. B. Bateman	16-23
1	ONR, Boston 10, Mass.	24
1	ONR, New York	25
2	ONR, Chicago	26-27
1	ONR, San Francisco	28
1	ONR, Pasadena	29
2	ONR, F.P.O. 100, New York	30-31
1	ONR, Washington, Attn: Mechanics Div. (Code 3800)	32
1	BuAer, Washington, Attn: Capt. W.S. Diehl (AER-23)	33
1	BuOrd, Washington, Attn: Code Re9	34
3	Wright Air Dev. Cen., Dayton, Mr. L.S. Wasserman, Flight Res. Lab	35-37
1	Air Res. and Dev. Command, Baltimore, Dr. T. Theodorsen	38
1	BuAer, Washington, Attn: Mr. J.S. Attinello	39
1	Director, David Taylor Mod. Basin, Washington, Attn: Dr. Brode	40
1	Air Materiel Command, Dayton, Air Doc. Div. (MCIDXR)	41
1	Ballistic Res. Lab., Aberdeen, Attn: Mr. R.H. Kent	42
1	Army Chemical Center, Attn: Dr. D. MacRae	43
1	NACA, Washington	44
1	NACA, Langley Aero. Lab., Langley Air Force Base, Va.	45
1	NACA, Ames Aero. Lab., Moffett Air Force Base, Calif.	46
1	NACA, Lewis Flight Propulsion Lab., Cleveland	47
1	Dr. Francis H. Clauser, Johns Hopkins Univ., Baltimore	48
1	Dr. Arnold Kuethe, University of Michigan, Ann Arbor	49
1	Dr. Hans W. Lippmann, GAL. CIT, Pasadena	50
1	Dr. G.B. Schubauer, Nat'l Bur. of Stds., Washington	51
1	Dr. R. J. Seeger, NOL, Silver Spring, Md.	52

	Universities and Research Organizations	
1	Atlantic Res. Corp., Alexandria, Attn: Dr. A.C. Scurlock	53
1	Polytech. Inst. of Brooklyn, Prof. R.P. Harrington	54
1	GAL, CIT, Pasadena, Attn: Dr. C. B. Millikin	55
1	Jet Prop. Lab., Cal. Tech., Pasadena, Attn: Librarian	56
1	Univ. of Calif., Berkeley, Attn: Dr. R. G. Folsom	57
1	Cornell Aero. Lab., Buffalo, Attn: Dr. J. V. Foa	58
1	Cornell Univ., School of Aero. Eng., Ithaca, Dr. W.R. Sears	59
1	Cornell Univ., School of Aero. Eng., Dr. A. Kantrowitz	60
1	Univ. of Del., Chem. Eng. Dept., Newark, Attn: Dr. K. Wohl	61
1	Harvard Univ., Dept. App. Phys. Engr. Sci., Attn: Prof. H. W. Emmons	62
1	Univ. of Illinois, Attn: Dr. H. F. Johnstone	63
1	Univ. of Illinois, Library	64
1	Univ. of Illinois, Eng. Library	65
1	Univ. of Iowa, Inst. Hydraulic Res., Attn: Dr. H. Rouse	66
1	Johns Hopkins Univ., Dept. Aero. Eng., Prof. S. Corrsin	67

Distribution List (Cont'd)

Copies	Universities and Research Organizations	Copy No.
1	Applied Phys. Lab., Silver Spring, Attn:Supervisor,Tech.Repts.	68
1	Arthur D. Little Co., Attn: Dr. R. V. Kleinschmidt	69
1	MIT, Gas Turbine Lab., Attn: Prof. A. H. Shapiro	70
1	MIT, Guided Missiles Comm., Attn: Supervisor, Tech. Repts.	71
1	MIT, Aero. Eng. Dept., Attn: Dr. J. C. Hunsaker	72
1	MIT, Chem. Eng. Dept., Attn: Prof. G. C. Williams	73
1	MIT, Math. Dept., Attn: Dr. C. C. Lin	74
1	MIT, Mech. Eng. Dept., Attn: Prof. J. H. Keenan	75
1	Univ. of Mich., Dept. Aero. Eng., Attn: Prof. E.W. Conlon	76
1	Princeton Univ., Aero. Eng. Dept., Attn: Prof. L. Lees	77
1	Princeton Univ., Gugg. Jet Prop. Lab., Attn: Prof. J.V. Charyk	78
1	Purdue Univ., Attn: Prof. M. J. Zucrow	79
1	Stanford Univ., Attn: Prof. P. A. Leighton	80
1	Univ. of Texas, Attn: Dr. M. J. Thompson	81
1	Univ. of Wichita, Aero. Eng. Dept., Attn: Mr. K. Razak	82
1	Appd. Phys. Lab., Silver Spring, Attn: Dr. F.N. Frenkiel	83
1	Purdue Univ., Dept. Chem., Met. Eng., Attn: Dr. E.W. Comings	84
1	Dr. T. Baron, Shell Devel. Co., Emeryville	85
1	Dr. A.E. Weller, Battelle Inst., Columbus	86
1	Dr. H. C. Hsu, Iowa Inst. of Hydraulic Res., Iowa City	87
1	Dr. S. I. Pai, Univ. of Maryland, College Pk.	88
1	Dr. Irvin Glassman, Guggenheim Lab., Princeton Univ.	89
1	Mr. Frank Kreith, Dept. Mech. Eng., Univ. of Cal., Berkeley	90
1	Nat. Res. Corp., Cambridge, Attn: Librarian	91
1	Mr. G. J. Maslach, Dept. Mech. Eng., Univ. Cal., Berkeley	92
1	Prof. R. W. McCloy, Univ. of Ill., Dept. Aero. Eng.	93
1	Prof. Paul Torda, Univ. of Ill., Dept. Aero. Eng.	94
1	Prof. Stillwell, Univ. of Ill., Dept. Aero. Eng.	95
1	Prof. H. H. Korst, Univ. of Ill., Dept. Mech. Eng.	96
1	Mr. S. F. Gilman, Univ. of Ill., Dept. Mech. Eng.	97
1	James Forrestal Res. Center, Princeton Univ., Attn: Librarian	98
1	Dr. Arnold Kivnick, Penn. Salt Mfg. Co., Philadelphia	99
1	Dr. L. G. Alexander, Oak Ridge, Tennessee	100

CROSS-WIRE PROBE FOR HOT-WIRE ANEMOMETER

J. E. Romano and H. F. Johnstone

An improved cross-wire probe for the study of turbulence in gas streams is described. It consists of two single-wire probes clamped together. The wires are oriented with the aid of a microscope and a haemocytometer in order to reduce the error due to the deviation of the wire angles. A method is presented for equalizing the time constants of the wires to reduce errors in compensation.

Introduction

Among the quantities which are of interest in studying the characteristics of turbulent flow are the intensities of the longitudinal and transverse components of the velocity fluctuations, the turbulent shear stress, and the uv correlation coefficient. Let u and v be the longitudinal and transverse components of the velocity fluctuations, respectively; ρ the density of the fluid; the primes refer to r. m. s. values and bars refer to time average values. Then the shear stress and the uv correlation coefficient are defined, respectively, by

$$\tau = -\rho\,\overline{uv} \qquad (1)$$

and

$$K_{uv} = \frac{\overline{uv}}{u'v'} \qquad (2)$$

The three quantities u', v', and $\overline{uv}$ can be measured with the hot-wire anemometer using a cross-wire probe placed in the fluid stream. The relations between these quantities and the response of the cross-wire probe are given by Schubauer and Klebanoff[1]. The fluctuating components of the voltages across the two

---

[1]Schubauer, G. B., and Klebanoff, P. S., A. C. R. No. 5K27, N. A. C. A. (1927).

---

wires for a constant-current hot-wire anemometer are given by

$$- e_1 = \delta_1 \frac{u}{\bar{U}} + \epsilon_1 \frac{v}{\bar{U}} \tag{3}$$

and

$$- e_2 = \delta_2 \frac{u}{\bar{U}} - \epsilon_2 \frac{u}{\bar{U}} \tag{4}$$

The sensitivities of the wires to velocity fluctuations are

$$\delta = \frac{(\bar{R} - R_a)^2}{2IR_a} F\bar{U}^{\frac{1}{2}} \tag{5}$$

and

$$\epsilon = \delta \cot \phi \tag{6}$$

The subscripts 1 and 2 refer to wires 1 and 2, respectively; $\bar{U}$ is the mean velocity, R the resistance of the wire at the wire temperature, R_a the resistance of the wire at the air temperature, I the current through the wire, and ϕ the angle between the wire and the axis of flow. F is the calibration coefficient in the equation

$$\frac{I^2\bar{R}}{\bar{R} - R_a} = A + F\bar{U}^{\frac{1}{2}} \tag{7}$$

A is an empirically determined constant for the wire.

Sources of Error

In developing the relations between the three turbulent quantities and the velocity fluctuations as given by Equations 3 and 4, Schubauer and Klebanoff[1] assumed that $\delta_1 = \delta_2$, $\phi_1 = \phi_2 = 45^\circ$, and that the attenuation in wire response due to the time constant of the wire, which is a measure of the lag of the wire to the fluctuating velocity signal, has been properly compensated. Kunstman[2] has shown that, although the sensitivies of the two wires

---

[2] Kunstman, R. W., Ph.D. Thesis in Chemical Engineering, University of Illinois (1952).

---

may be different, little error will be incurred if the turbulent quantities

are calculated from the following equations

$$\frac{u'}{U} = \frac{(-e_1 - e_2)'}{\delta_1 + \delta_2} - \frac{(\delta_1 - \delta_2)}{(\delta_1 + \delta_2)} \frac{(-e_1 + e_2)'}{(\delta_1 + \delta_2)} \qquad (8)$$

$$\frac{v'}{U} = \frac{(-e_1 + e_2)'}{\delta_1 + \delta_2} - \frac{(\delta_1 - \delta_2)}{(\delta_1 + \delta_2)} \frac{(-e_1 - e_2)'}{(\delta_1 + \delta_2)} \qquad (9)$$

$$\frac{\overline{uv}}{U^2} = \frac{1}{4}\left(\frac{-e_1'}{\delta_1}\right)^2 - \frac{1}{4}\left(\frac{-e_2'}{\delta_2}\right)^2 \qquad (10)$$

The error in ϵ caused by deviations of ϕ from 45°, however, may be considerable

Table I

Errors in Wire Sensitivity ϵ caused by

Deviations of ϕ from 45°

ϕ	Per Cent Error	ϕ	Per Cent Error
40	+ 16.1	46	- 3.6
41	+ 13.1	47	- 7.2
42	+ 10.1	48	-11.1
43	+ 6.8	49	-15.0
44	+ 3.4	50	-19.2

as shown in Table I.

Appreciable error will also result if the time constants of the wires are not equal. Dryden and Kuethe[3] have shown that the actual value of the

[3] Dryden, H. L., and Kuethe, A. M., T. R. 320, N. A. C. A. (1929).

fluctuating voltage of an anemometer wire is lower in amplitude than predicted by Equations 3 and 4 because of the slow response of the wire resulting from its mass. The ratio of the actual r. m. s. voltage to that for a wire with zero mass is given by

$$\frac{e_a'}{e_t'} = \frac{1}{\sqrt{1 + 4\pi^2 f^2 M^2}} \qquad (11)$$

Here, M is the time constant of the wire, and f is the frequency of the fluctuating velocity being measured. To compensate for this attenuation, a circuit may be introduced in the hot-wire anemometer with the inverse characteristics of the right hand side of Equation 11. Since the attenuation depends on the frequency, a signal sent to the wire containing many frequencies (e.g., a square wave signal) will be distorted. The compensator is adjusted until such a signal remains undistorted. If the response of the two wires must be measured simultaneously, i.e., when measuring the sums or differences of the instantaneous voltages of the two wires, the time constants of both wires must be the same since the compensator must compensate for the attenuation of both wires simultaneously.

Adjustment of Wire Angles

Deviations from the assumed condition $\phi_1 = \phi_2 = 45°$ arise from two sources: (1) the deviation of the angle between the two wires from 90° when constructing the cross-wire probe, and (2) the error in alignment of the probe axis with the axis of flow. Because of the difficulty of accurately controlling the angle between the wire and the axis of the wire holder during the construction of a probe, a cross-wire probe with both wires fixed on one holder does not lend itself to accurate control of the wire angles. A system of two single-wire probes is more flexible.

The properties of the wire and supports for each single-wire probe were chosen from a consideration of a balance of their limiting characteristics as

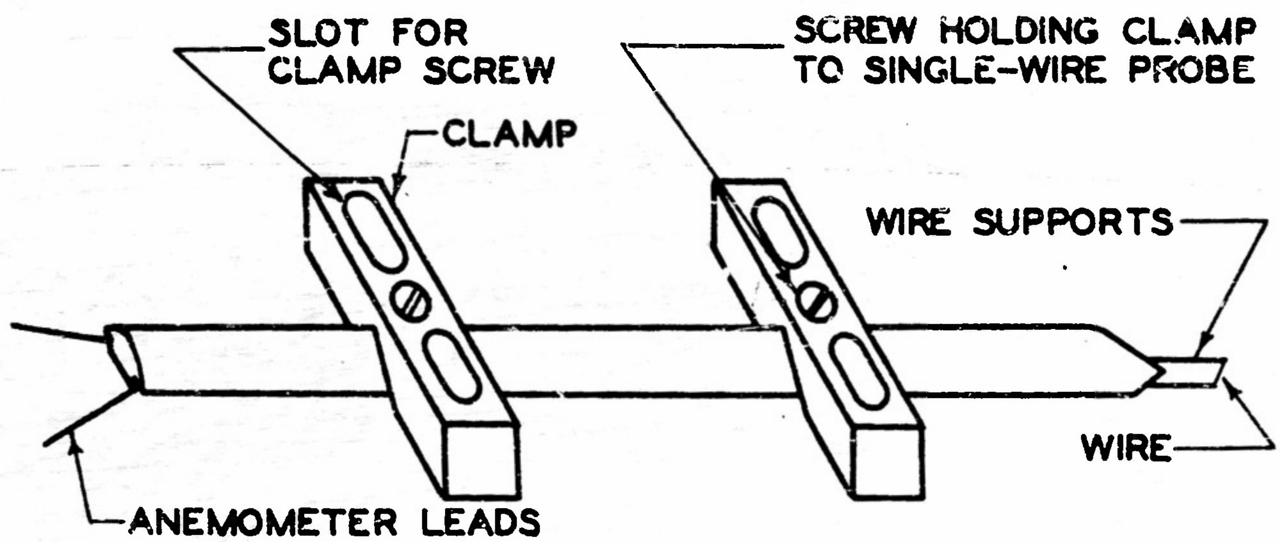
discussed by Zignen[4]. The probe wire used was 0.000139 inch diameter

[4]Zignen, B. G. van der Hegge, Applied Sci. Res. A2, 351 (1951).

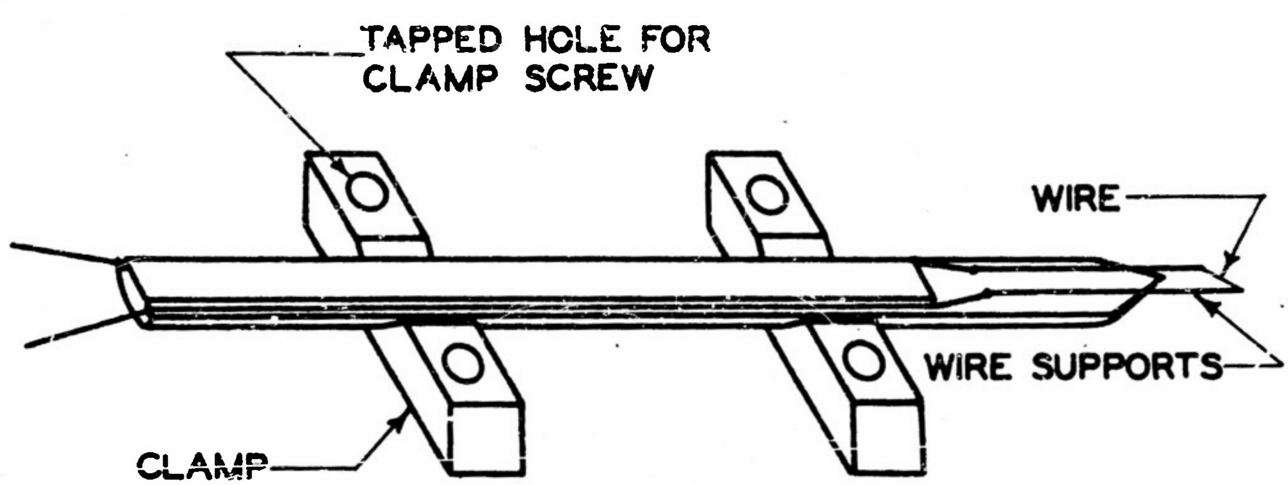
tungsten, and the wire supports were No. 12 nickel plated Boye embroidery needles spaced 1/16 inch apart. The wire was soldered to the supports at approximately 45° to the probe axis by means of a jig, and cleaned with hot benzene to remove excess flux. If any foreign particles were detected on the wire under a binocular microscope,which could not be removed with repeated washings with hot benzene or with blasts of air, the wire was removed and the construction was repeated with a new wire.

The two single-wire probes were put together to form a cross-wire probe and held firmly by means of clamps as shown in Figure I. The clamps were first fastened loosely and the top probe adjusted until the wires were at exactly 90° to each other with their midpoints in a line perpendicular to the horizontal. A binocular microscope and a haemocytometer were used to obtain the correct orientation as shown in Figure 2. The clamp screws were tightened and the distance between the wires observed with a second microscope. This distance could be adjusted with paper shims between the clamps. The error in orientation of a wire with another wire could thus be reduced to less than $\pm$ 0.5°. This was possible because the diameter of the wire and the thickness of the cross-hatches on the haemocytometer were comparable in size, and because the length-to-diameter ratio of the wire was 635.

In order to reduce the error in alignment of the probe axis with the axis of flow, the binocular microscope and the haemocytometer were also used to orient the cross-wire probe in the gas stream.The grid lines on the haemocytometer were



TOP HALF OF CROSS-WIRE PROBE



BOTTOM HALF OF CROSS-WIRE PROBE

FIGURE I. CROSS-WIRE PROBE

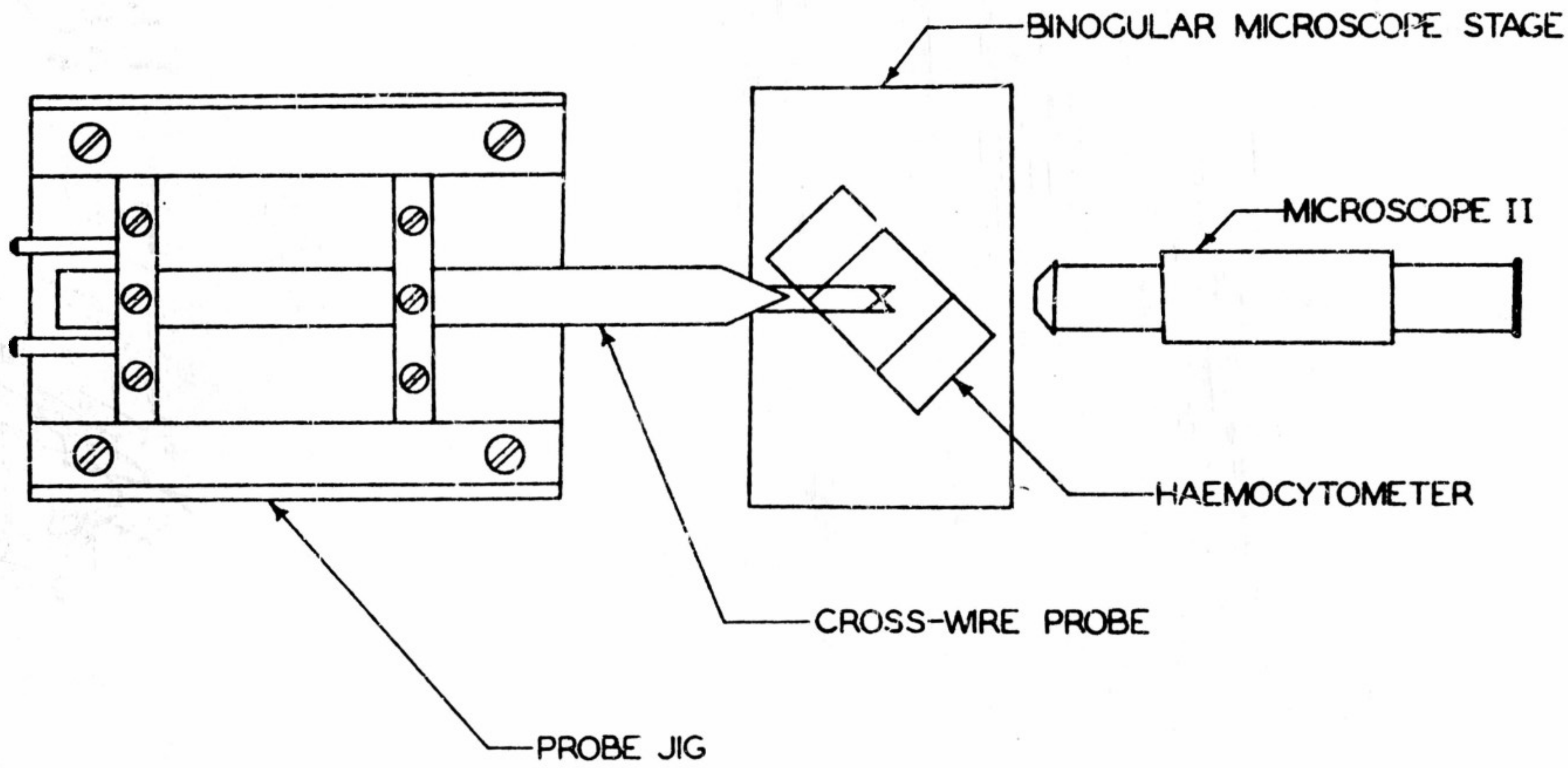


FIGURE 2. ASSEMBLY OF CROSS-WIRE PROBE

first fixed at 45° to the axis of flow by bringing the face of the haemocytometer in line with a jig (Figure 3), which had been adjusted to 45° with the axis of flow. The binocular microscope was then used to orient the wires to coincide with the grid lines on the haemocytometer.

Equalization of Wire Time Constants

Corrsin[5] has shown that the time constant for a wire is given by

---

[5]Corrsin, S., T. N. 1864, N. A. C. A. (1949).

$$M = \frac{4.2ms\,(\bar{R}-R_a)}{I^2\,R_a R_o} = \frac{4.2ms}{R_a R_o}\,P \qquad (12)$$

where R_o is the resistance of the wire at 0°C., m the mass of the wire, s its specific heat, and the other variables are as defined previously. For a given wire, the only terms in Equation 12 that can be varied in order to adjust M to a predetermined value (i.e., to match the value of M for a second wire) are contained in the term P, viz., the current I and the heat transfer potential $(\bar{R} - R_a)$.

Figure 4 shows the observed variation of the parameter P with the wire temperature for one of the single-wire probes studied. The data indicate that a change of wire temperature of 120°C. produces approximately a thirty percent change in the parameter, and, therefore, a thirty percent change in the wire time constant. Thus, by varying the current through one of the two wires, the time constant of this wire can be adjusted to match the time constant of the other wire.

The exact procedure for determining the correct compensator setting for

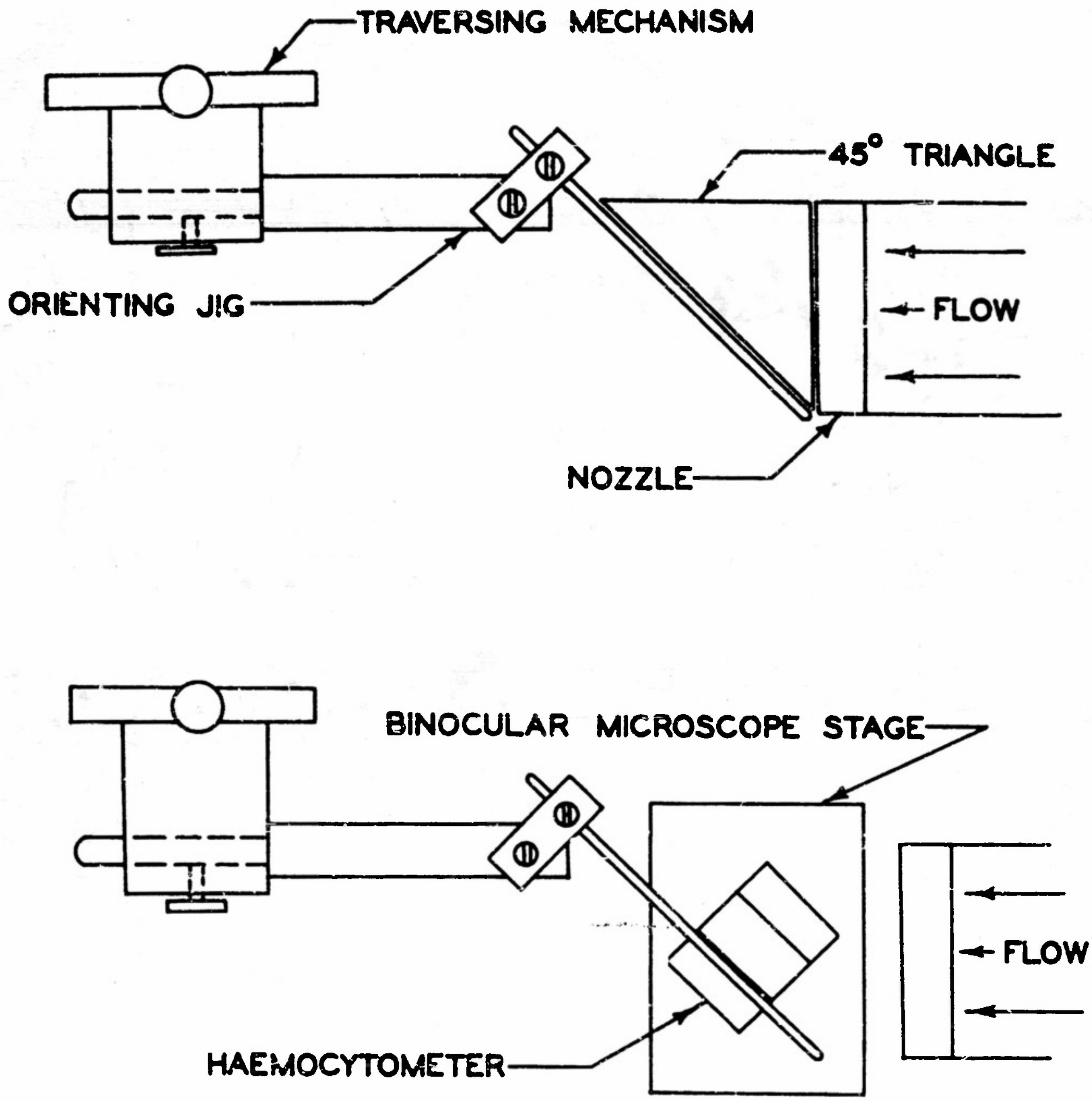


FIGURE 3. ORIENTATION OF HAEMOCYTOMETER

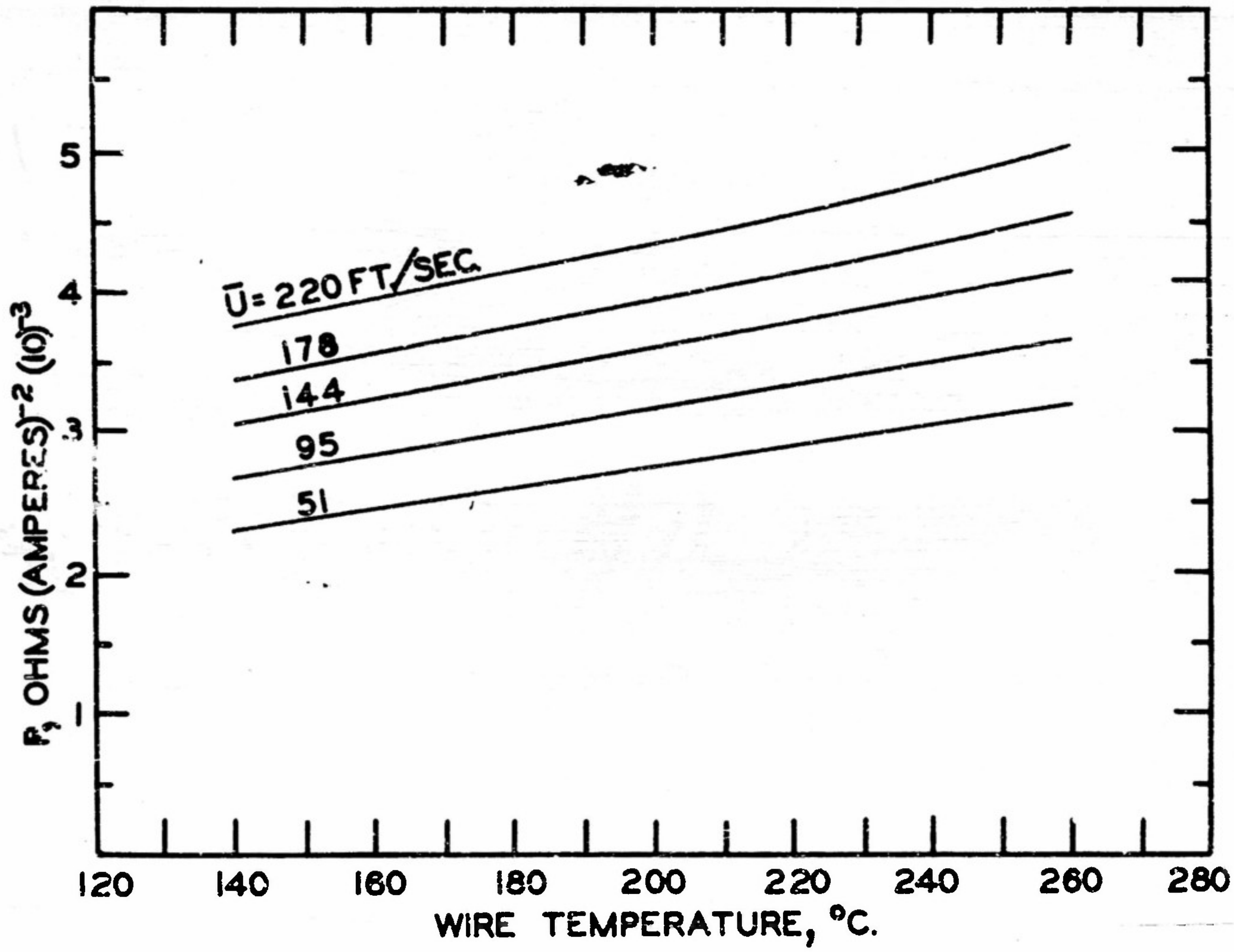


FIGURE 4. VARIATION OF TIME CONSTANT PARAMETER P WITH WIRE TEMPERATURE

both wires is as follows. A square wave signal is first sent to one of the two wires. The temperature of this wire is then set equal to the temperature used in obtaining the calibration coefficients of this wire (see Equation 7). This is accomplished by adjusting a direct current through the wire until the wire attains a resistance, $\overline{R}_1$, corresponding to the calibration temperature. The compensator is then adjusted until the square wave signal is no longer distorted. Maintaining the compensator at this setting, the square wave signal is then sent to the second wire and the direct current through the second wire is adjusted until the signal is no longer distorted. The resistance of the second wire under these conditions, $\overline{R}_2$, is then measured. For the given flow conditions, if the first wire is operated at $\overline{R}_1$ and the second wire operated at $\overline{R}_2$, the two wires will have the same time constant and a single setting of the compensator will correctly compensate for the attenuation by each wire when measuring the response of the two wires simultaneously.

The temperature at which the first wire is operated for turbulence measurements is the temperature at which it was calibrated. Since the current through the second wire is adjusted until its time constant equals that of the first wire, this wire will be operated at a temperature different from that used for its calibration. This will affect the value of the calibration coefficient in Equation 7, and, consequently, the wire sensitivity as defined in Equation 5. A relationship between the calibration coefficient and the wire temperature is required. In order to find this relationship, several probes were calibrated separately at different wire temperatures as shown in Figure 5. The observed values of F_2/F_1 are given in Table II.

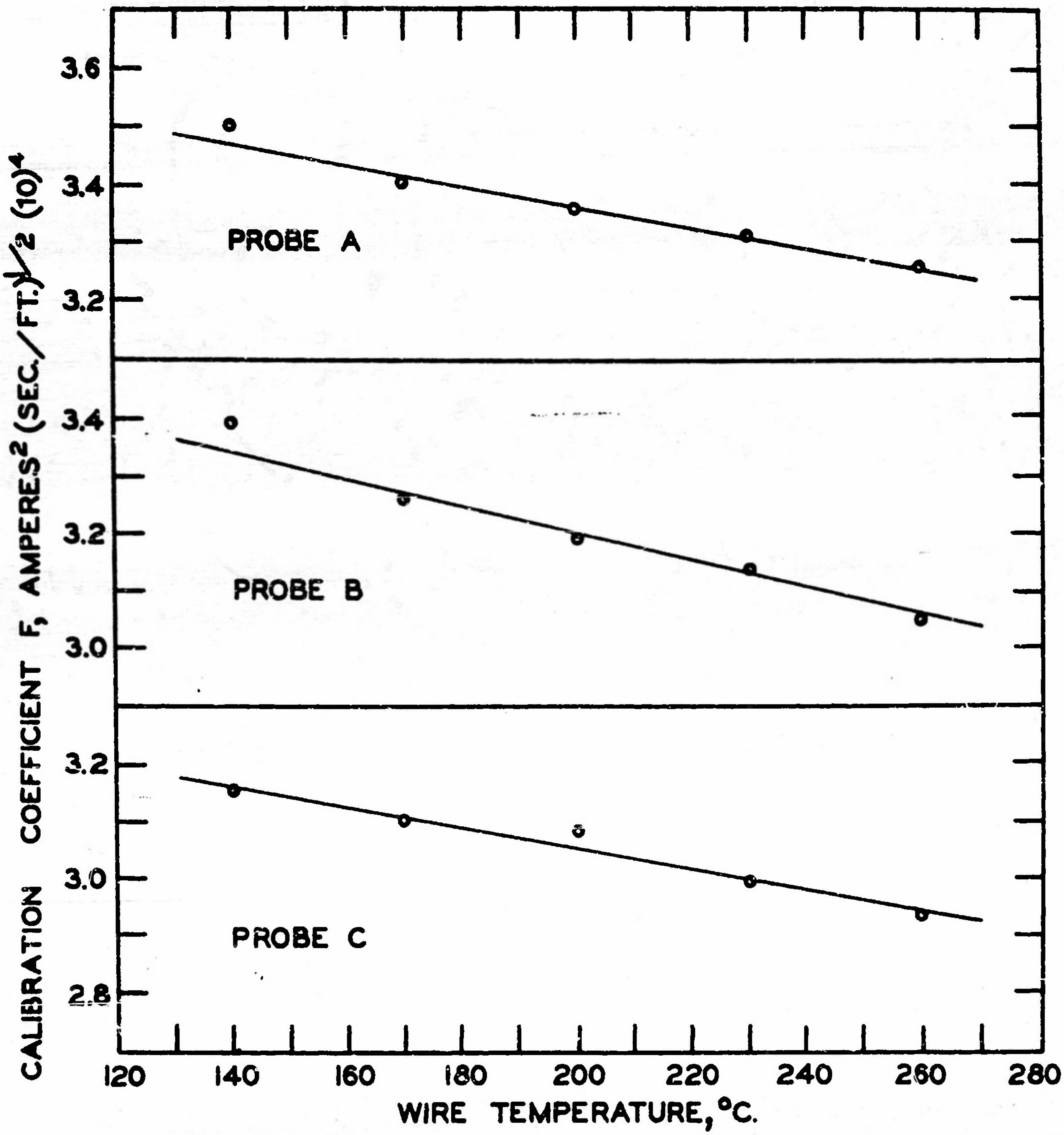


FIGURE 5. VARIATION OF CALIBRATION COEFFICIENT F WITH WIRE TEMPERATURE

Table II

Effect of Wire Temperature on Calibration Coefficient F

Reference wire temperature = 150°C.

Probe	t_w °C.	$(T_1/T_2)^{\frac{1}{2}}$	F_2/F_1 Observed
A	200	0.963	0.971
	230	0.950	0.955
	260	0.932	0.941
B	200	0.963	0.964
	230	0.950	0.944
	260	0.932	0.923
C	200	0.963	0.970
	230	0.950	0.954
	260	0.932	0.937

The simple equation

$$\frac{F_2}{F_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{2}} \qquad (13)$$

represents the effect of the temperature on F within one percent. Here, T denotes the arithmetic average of the wire temperature and the air temperature in °K. The new value of the calibration coefficient F resulting from a change in its temperature can be determined with this equation. Thus, the time constant of two wires can be equalized by adjusting the current through one of the wires until a single setting on the compensator suffices.

This report has been submitted for publication in Review of Scientific Instruments.